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Application of time-dependent suction to free convection laminar flow

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The boundary layer equations for the laminar flow past a porous vertical wall has been discussed for the free convection when the suction velocity is an oscillatory function of time. Expressions for the velocity and temperature distributions have been calculated in the non-dimensional forms. From these equations, the rate of heat transfer from the wall to the fluid, Nusselt number and skin-friction have been calculated. It is found that the rate of heat iransfer from the wall to the fluid, decreases as the suction velocity increases. The phase angle of the skinfriction is also found to decrease with increasing the unsteady part of the suction velocity. Giaphically the variations for phase angle, steady part of velocity distribution the skin friction have been shown in some cases when the frequency of fluctuations are small or large. For large frequency of oscillations, he skin friction is found to lag behind the wall velocity fluctuations. This phase angle s found to decrease as the suction velocity increases.

1. INTRODUCTION

The unsteady two-dimensional laminar flow was considered by Lighthill (1954) for the velocity and thermal boundary layers. He has analyzed mathematically the equations of motion and energy when the velocity of the on-coming flow relative to the body oscillates in magnitude but not in direction. (Messiha, 1966) has considered unsteady oscillatory flow past an infinite flat plate and calculated the expressions for the velocity, temperature and skin-friction for small and large frequency of oscillations. It has been assumed (Messiha, 1966) that the suction velocity and free stream velocity and free stream velocity are both functions of time. It has been concluded there that the effect of increasing the suction velocity is that the amplitude of skin frictian is increased and the phase is decreased. For small value of the frequency of fluctuation, it is given (Messiha, 1966) that the wall temperature decreases for increasing the suction velocity. Lal (1966) has considered the free convection problem when the suction velocity depends on time and expressians for skin and rate of heat transfer are deduced.

In the present paper, the author has considered the problem of Messiha (1966) for free convention laminar flow when the dissipation tern is neglected. The suction velocity is taken as $Vo[1+\epsilon Ae^{i\omega t}]$ where Vo is constant mean velocity and $\epsilon A \leqslant 1$. For A=O, the problem is reduced to steady suction velocity. Expressions for u (y, t), temperature, rate of heat transfer from the wall to the liquid, skin-friction have been calculated and interesting results are obtained regarding the dependence of the skin-friction and the rate of heat transfer from the boundary to fluid, on the fluctuating part of the suction velocity. The expressions are expanded for large and small frequency of oscillations and graphically some variations are represented.

2. Fundamental equations

Let $\bar{x}$-axis be taken along the vertical plate and $\bar{y}$-axis perpendicular to it. The equations which describe the unsteady free convection flow of a viscous incompressible fluid for the present problem are

$$\frac{\delta \bar{u}}{\delta \bar{t}} + \bar{v}\,\frac{\delta \bar{v}}{\delta \bar{y}} = g\beta\,(\,\bar{T} - \bar{T}_{\infty}\,) + \nu\,\frac{\delta^2 \bar{u}}{\delta \bar{y}^2} \qquad \ldots 1.1$$

$$\frac{\delta \bar{v}}{\delta \bar{y}} = 0 \qquad \ldots 1.2$$

$$\frac{\delta \bar{v}}{\delta \bar{t}} = -\,\frac{1}{\rho}\,\frac{\delta \bar{p}}{\delta \bar{y}} \qquad \ldots 1.3$$

$$\frac{\delta \bar{T}}{\delta \bar{t}} + \bar{v}\,\frac{\delta \bar{T}}{\delta \bar{y}} = K\,\frac{\delta^2 \bar{T}}{\delta \bar{y}^2} \qquad \ldots 1.4$$

where $\bar{u}$, $\bar{v}$ are velocity components, ρ the density, g the acceleration due to gravity, β the coefficient of volume expansion, $\bar{t}$ time, $\bar{p}$ the pressure; v the kinematic coefficient of viscosity, $\bar{T}$ the temperature, $\bar{T}_{\infty}$ the temperature at infinity, K the thermal deffusivity. From (1.2), it is clear thrt $\bar{v}$ is a function $\bar{t}$ only. In a thin boundary layer, $\frac{\delta \bar{p}}{\delta \bar{y}}$ is very small and so if $-v_0$ be the steady suction velocity, we replace $\bar{v}$ by

$$\bar{v} = v_0\,[\,1 + \epsilon A\, e^{i\bar{\omega}\bar{t}}] \qquad \ldots 1.5$$

where

$$\epsilon A \leqslant 1,$$

The above equations are reduced into non-dimensional froms by the substitutions of

$$\left.\begin{array}{l} y = \frac{\bar{y}}{L}\ ,\ t = \frac{\nu \bar{t}}{L^2}\ ,\ u = \frac{\bar{u}L}{|v_o|\,\nu}\ ,\ v = \frac{\bar{v}L}{\nu} \\ T = g\beta L^3\,(\ \bar{T} - \bar{T}_\infty\ )\,/\,|\,v_o\,|\,\nu^2,\ p = \bar{p}L^2\,/\,\rho\nu^2\,|\,v_o\,|\ ,\ \omega = \frac{L\bar{\omega}}{\nu} \end{array}\right\} \quad \ldots 1.6$$

where $\bar{T}_\infty$ is reference temperature and L is characteristic length. Equations 1.1 and 1.4 are replaced by

$$\frac{\delta u}{\delta t} - [\,1 + \epsilon A\, e^{i\omega t}\,]\frac{\delta u}{\delta y} = T + \frac{\delta^2 u}{\delta y^2} \quad \ldots 1{\cdot}7$$

$$\frac{\delta T}{\delta t} - [\, + \epsilon A\, e^{i\omega t}\,]\ \frac{\delta T}{\delta y} = \frac{1}{\sigma}\ \frac{\delta^2 T}{\delta y^2} \quad \ldots 1.8$$

where σ is the Prandtl number. The boundary conditions are

$$\left.\begin{array}{l} t < 0 : u = v = T = o \text{ for } y \geqslant 0 \\ t \geqslant 0 : u = o,\ v = -\ v_o\,(\ t\ ),\ T = \theta\,(\ t\ ) \text{ for } y = 0 \\ y = \infty : u\,(\ \infty, t\ ) = 1,\ T = 0 \end{array}\right\} \quad \ldots 1.9$$

2. Solutions of equations

To solve the equations 1.7 and 1.8, with above boundary conditions, we assume

$$u\,(y,t) = F_o(y) + F_1(y)\epsilon e^{i\omega t} + F_2(y)\epsilon^2 e^{2i\omega t} + \ldots + F_n(y)\epsilon^n e^{in\omega t} \quad \ldots 2.1$$

$$T\,(y,t) = T_o(y) + T_1(y)\epsilon e^{i\omega t} + T_2(y)\epsilon^2 e^{2i\omega t} + \ldots + T_n(y)\epsilon^n e^{in\omega t} \quad \ldots 2.2$$

where ϵ is small parameter such that $\epsilon^3, \epsilon^4, \epsilon^5, \ldots \epsilon^n$ are neglegibly small quantities. If, we give the wall temperature as cos ωt, cos $2\omega t$, ... we consider real parts of equations 2.1 and 2.2.

Substituting 2.2 into 1.8 and equating the coefficient of ϵ^n and harmonic terms, we have

$$in\,\omega\, T_n - A\, T'_{n-1} = \frac{T''_n}{\sigma} \quad \ldots 2.3$$

where

$$n = 0, 1, 2, \;\ldots$$

The boundary conditions for T_n are reduced to

$$\left.\begin{array}{l} y = 0 : T_o = T_o(0) = \theta_o \text{ (say)},\ T_1(0) = \theta_1, \ldots\ T_n(0) = \theta_n \\ y = \infty : T_o = T_1 = T_2 = \;\ldots\; = T_n = 0 \end{array}\right\} \quad \ldots 2.4$$

Solving 2.3 with the boundary conditions from the set 2.4, we have

$$T_o(y) = \theta_o \exp(-\sigma y) \quad \ldots 2.5$$

$$T_1(y) = \theta_1 e^{-hy} + \frac{i\theta_o \sigma A}{\omega}\left[e^{-\sigma y} - e^{-hy}\right] \quad \ldots 2.6$$

$$T_2(y) = \theta_2 e^{-my} + \frac{Ah\sigma\left[\theta_1 - \dfrac{i\theta_o \sigma A}{\omega}\right]}{h^2 - h\sigma - 2\, i\omega\sigma}\left[e^{-hy} - e^{-my}\right] - \frac{\theta_o \sigma^2 A^2}{2\,\omega^2}\left[e^{-\sigma y} - e^{-my}\right] \ldots 2.7$$

$$\text{where, } h = \frac{\sigma}{2}\left[1 + \sqrt{1 + 4i\omega/\sigma}\,\right],\ m = \frac{\sigma}{2}\left[1 + \sqrt{1 + 8i\omega/\sigma}\,\right] \quad \ldots 2.8$$

Using equaitons 2.1 and 2.2 into 1.7, we have following relation from equating the coefficients of ϵ^n and harmonic terms,

$$i\omega n - F_n F'_n - AF'_{n-1} = T_n + F_n'' \quad \ldots 2.9$$

where $n = 0, 1, 2, \ldots$

Solving above equation and the following boundary conditions for F_n

$$\left.\begin{array}{l} y = 0 : F_o = F_1 = F_2 = \ldots = F_n = 0 \\ y = \infty : F_o = 1,\ F_1 = F_2 = \ldots F_n = 0 \end{array}\right\} \quad \ldots 2.10$$

we have for F_o F_1 and F_2

$$F_o(y) = 1 - e^{-y} + a\left(e^{-\sigma y} - e^{-y}\right) \quad \ldots 2.11$$

$$F_1(y) = \frac{A(1+a)}{iw}\left[e^{-y} - e^{-my}\right] + b\left[e^{-\sigma y} - e^{-my}\right] + c\left[e^{-hy} - e^{-my}\right] \quad 2.12$$

$$F_2(y)=\frac{e^{-my}}{m^2-m-2iw}\left[-\theta_2+\frac{Ah\sigma\left(\theta_1-\frac{i\theta_0\sigma A}{\omega}\right)}{h^2-h\sigma-2i\omega\sigma}-\frac{\theta_0\sigma^2A^2}{2w^2}\right]$$

$$-\frac{e^{-m_1y.}}{m_1^2-m_1-2iw}\left[\frac{m_1A^2(1+a)}{iw}+m_1Ab+m_1Ac\right]+\frac{\sigma Ae^{-\sigma y}}{\sigma^2-\sigma-2iw}\left(-b+\frac{\theta_0\sigma A}{2w^2}\right]$$

$$+\frac{Ahe^{-hy}}{h^2-h-2iw}\left[c-\frac{a\left(\theta_1-\frac{i\theta_0\sigma A}{w}\right)}{h^2-h\sigma-2i\omega\sigma}\right]+\frac{A^2(1+a)}{2w^2}e^{-y}+c_1e^{-Dy} \quad \ldots 2.13$$

where the constant of integration C_1, is easily obtained by using the boundary condition at $y=.0$ The values of the constants D, a, b and c are

$$\left.\begin{aligned} &D=\tfrac{1}{2}\left[1+\sqrt{1+\sigma i\omega}\right],\ a=\frac{\theta_0}{\sigma-\sigma^2},\\ &b=\frac{A\sigma\left(a-\frac{i\theta_0}{\omega}\right)}{\sigma^2-\sigma-i\omega},\ C=\left(\frac{i\theta_0\sigma A}{\omega}-\theta_1\right)/(h^2-h-i\omega)\\ &m_1=\tfrac{1}{2}[1+\sqrt{1+4iw}]\end{aligned}\right\} \quad \ldots 2.14$$

3. Discussions

If q, be the rate of heat addition from the plate to the fluid, we have

$$q=-k\left[\frac{\delta\bar{T}}{\delta\bar{y}}\right]\bar{y}=0$$

$$=\frac{k\,v_0\nu^2}{g\beta L^4}\left\{\theta_0\sigma+\epsilon e^{i\omega t}\left[\theta_1h+\frac{i\theta_0\sigma A}{\omega}(\sigma-h)\right]+\ldots\ldots\right\} \quad \ldots 3.1$$

after substitutions and simplifications.

And if N be the Nusselt number, we have

$$N=\frac{Lq}{k(\bar{T}_\omega-\bar{T}_\alpha)}=\frac{\theta_0\sigma+\epsilon e^{i\omega t}\left\{\theta_1h+\frac{i\theta_0\sigma A}{\omega}(\sigma-h)\right\}+\ldots}{\theta_0+\theta_1\epsilon e^{i\omega t}+\theta_2\epsilon^2e^{2i\omega t}+\ldots} \quad \ldots 3.2$$

where $\bar{T}_\omega$ is the wall temperature. Since $h>\sigma$, we see that q and N decrease as A increases or the suction velocity increases.

If T_0 be the skin-friction, we have

$$T_0 = \mu \left[\frac{\delta \bar{u}}{\delta \bar{y}} \right] \bar{y}=0$$

$$= \frac{\mu^2 v_0}{L^2 \rho} \left[1+a(1-\sigma)+\epsilon e^{i\omega t} \left\{ b\,(m_1-\sigma)+c(m_1-h)-\frac{iA\,(1+a)}{w}(m_1-1) \right\} + \ldots \right] \quad 3.3$$

if the terms having ϵ^2, ϵ^3 are not considered.

The skin-friction is obtained in terms of b and c which are complex quantities. Hence to combine ωt with phase angle, we have to separate real and imaginary part.

Case (i) if ω *is small*

From the expressions for F_0 and T_0, we see that these are independent of ω. Hence we have deduced the expressions of F_1 and T_1 for small or large frequency of oscillations. For small values of ω, we have

$$m \simeq \left(\sigma + \frac{4\omega^2}{\sigma} \right) + i \left(2\,\omega - \frac{16\,\omega^3}{\sigma^2} \right) + 0\,(\omega^4) \quad 3.4$$

$$h \simeq \left(\sigma + \frac{\omega^2}{\sigma} \right) + i \left(\omega - \frac{2\,\omega^3}{\sigma^2} \right) + 0\,(\omega^4) \quad 3.5$$

which may be written as for example

$$m = m_r + i\,m_i \quad 3.6$$

where $m_r = \left(\sigma + \frac{4\,\omega^2}{\sigma} \right)$, $m_i = \left(2\,\omega - \frac{16\,\omega^3}{\sigma^2} \right)$ if terms of order ω^4 are neglected. Similarly, we use

$$h = h_r + i\,h_i \quad \ldots\ 3.7$$

where h_r and h_i obtained as in the case of m_r and m_i. In the same manner, we have

$$b = b_r + i\,b_i$$

$$= \left[\left(\frac{A}{\sigma-1} + \frac{A\,\theta_o}{\sigma(\sigma-1)^2}\right) + \omega^2\left(\frac{A\,\theta_o}{\sigma^3(\sigma-1)^4} - \frac{A}{\sigma^2(\sigma-1)^3}\right)\right]$$

$$+ i\left[-\frac{A\,\theta_o}{\omega(\sigma-1)} + \frac{A\,\omega}{\sigma(\sigma-1)^2}\left(1 + \frac{\theta_o}{\sigma(\sigma-1)}\right)\right] \qquad 3.8$$

$$h^2 - h - i\omega = \left[\sigma(\sigma-1) + \frac{\sigma-1}{\sigma}\omega^2\right] + i\left[2\omega(\sigma-1) - \frac{2\omega^3}{\sigma^2}(\sigma-1)\right] \qquad 3.9$$

and

$$C = Cr + iC_i$$

$$= \frac{1}{\sigma(\sigma-1)}\left\{\left[2\theta_o A - \theta_1 + w^2\left(\frac{5\theta_1}{\sigma^2} - \frac{6\theta_o A}{\sigma^2}\right)\right]\right.$$

$$\left. + i\left[\frac{\theta_o\sigma A}{w} + w\left(\frac{2\theta_1}{\sigma} - \frac{5\theta_o A}{\sigma}\right) + w^3\left(-\frac{6\theta_1}{\sigma^3}\right)\right]\right\} \qquad \ldots 3.10$$

If w^2 and high er products of w are neglected, we have

$$u(y,t) = 1 - e^{-y} + \frac{\theta_o}{\sigma(1-\sigma)}\left[e^{-\sigma y} - e^{-y}\right] + \epsilon e^{i\omega t}\left[F_{1r} + iF_{1i}\right] \qquad \ldots 3.11$$

where

$$F_{1r} = \frac{A\,e^{-\sigma y}}{\sigma(\sigma-1)}\left[y(\theta_o\sigma - \theta_o + \sigma^2 - \sigma)\right] \qquad \ldots 3.12$$

and

$$F_{1i} = \frac{Aw\theta o(2\sigma-1)e^{-\sigma y}}{\sigma(\sigma-1)^2}\left[y + \frac{y^2(\sigma-1)}{2(2\sigma-1)}\left\{2 + \frac{2(\sigma-\sigma^2)}{\theta_o} - \sigma\right\}\right]$$

$$+ \frac{A}{w}\left[\frac{\theta_o + \sigma - \sigma^2}{\sigma^2 - \sigma}\left(e^{-y} - e^{-\sigma y}\right)\right] \qquad \ldots 3.13$$

Thus if only real parts are considered, we have from above equations,

$$u(y,t) = 1 - e^{-y} + \frac{\theta_o}{\sigma(\sigma-1)}\left(e^{-y} - e^{-\sigma y}\right) + \epsilon\,|\,F_1\,|\cos(\omega t + \alpha) \qquad \ldots 3.14$$

where

$$\left.\begin{aligned} |F| &= \sqrt{F_{1r}^2 + F_{1i}^2} \\ \alpha &= \tan^{-1}\left(\frac{F_{1i}}{F_{1r}}\right) \end{aligned}\right\} \qquad ...3.15$$

Thus the phase angle of the velocity distribution inside the boundary layer leads by an angle α.

The motion becomes independent of time if

$$\alpha = \pi\left(n + \tfrac{1}{2}\right) - \omega t,\ n = 0, 1, 2, \qquad ...3.16$$

Also, we have $\alpha = \pi/2$ for $\theta_0 = -\sigma$. From above equations, we see that the unsteady part of $u(y, t)$ increases by increasing the value of A, on which the suction velocity depends. Thus increasing the suction velocity, we see that the unsteady part of the velocity distribution inside the boundary layer increases. The variations of steady part of $u(y, t)$ with y, have been shown in figure 1.

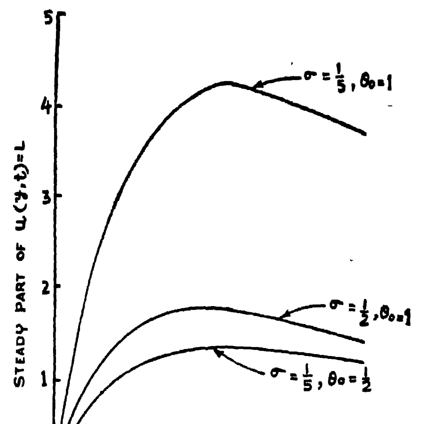

Figure 1. Graph between $1-e^{-y} + \theta_0(\sigma - 1)(e^{-y} - e^{-\sigma y})$ and y for given θ_0 and σ,

The variations of other parameters such as F_{1r}, F_{1i}, F_1 and α may also similarly be represented graphically. The graphical representations in such cases are done in other cases.

Using the expansions for h from equation 3.5 into 2.2,

$$T(y,t) = \theta_0 \exp(-\sigma y) + \epsilon e^{i\omega t}[T_{1r} + i T_{1i}] \qquad ...3.17$$

when ϵ^2 and other small terms are neglected. The expressions for T_{1r} and T_{1i} are given by

$$\left.\begin{aligned} T_{1r} &= e^{-\sigma y}(\theta_1 - \theta_0 \sigma A y) \\ T_{1i} &= \omega e^{-\sigma y}\left[\frac{\theta_0 \sigma A y^2}{2} - y\theta_1\right] \end{aligned}\right\} \qquad ...3.18$$

Thus we may easily write

$$T(y,t) = \theta_0 e^{-\sigma y} + \epsilon \mid T_1 \mid \cos(\omega t + \gamma) \qquad ...3.19$$

where,

$$\left.\begin{aligned} \mid T_1 \mid &= \sqrt{T_{1r}^2 + T_{1i}^2} \\ \gamma &= \tan^{-1}\frac{T_{1i}}{T_{1r}} \end{aligned}\right\} \qquad ...3.20$$

The variation of angle γ with y has been shown graphically in figure 2 for given σ, A, θ_0, θ_1, ω. The value of γ on the wall is $\pi/_2$ if $\theta_1 = 0$. From the figure (when $A = \theta_0 = \frac{1}{2}$, $\sigma = 1$, $\theta_1 = \frac{1}{4}$) we see that if ω increases, the value of phase angle, γ also increases with respect to y. The angle γ becomes negative between $y = 0$ and $y = 1$ which implies that temperature distribution inside the boundary layer lags behind the wall fluctions between $y = 0$ and $y = 1$. Between $y = 1$ and $y = 2$, this angle is positive and then temperature distribution leads by an angle γ for $1 \leqslant y \leqslant 2$.

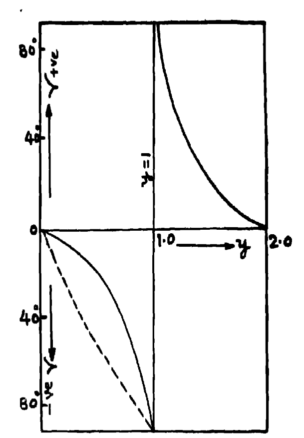

Figure 2. ————, $\omega = 0.5$; ...., $\omega = 2.0$

From equations 3.5 and 3.1, we have

$$q = \frac{K v_0 \nu^2}{g\beta L^4}\left\{ \theta_0\sigma + \epsilon e^{i\omega t}\left[\theta_1\sigma + \theta_0\sigma A + i\,\omega\theta_1 \right] + \dots \right\} \qquad \dots 3.21$$

which considering the real parts may be written as

$$q = \frac{K v_0 \nu^2}{g\beta L^4}\left\{ \theta_c\sigma + \epsilon \;|\,B\,|\; \cos(\omega t + \Phi) + \dots \right\} \qquad \dots 3.22$$

$$\left.\begin{aligned} &\text{where } B = B_r + i\,B_i = \sigma(\theta_1 + \theta_0 A) + i(\omega\,\theta_1) \\ &\text{and } \phi = \tan^{-1}\cdot\frac{B_i}{B_r} \end{aligned}\right\} \qquad \dots 3.23$$

By an inspection of equations of set 3.23, we see that angle Φ will be positive and thus the rate of heat transfer has a lead over the surface temperature fluctuations. The variations of angle Φ with ω has been drawn in figure 3 for various values of A when $\theta_0 = \theta_1 = \frac{1}{2}$, $\sigma = 2$. From figure 3, we see that this angle Φ is positive and increases as ω increases. The increment in Φ is large upto $\omega = 1.5$ and from $\omega = 1.5$ to 3.5 slow and for $\omega > 3.5$ it is very slow. As A increases, the angle Φ is found to decrease. If A increases, we see that the suction velocity increases.

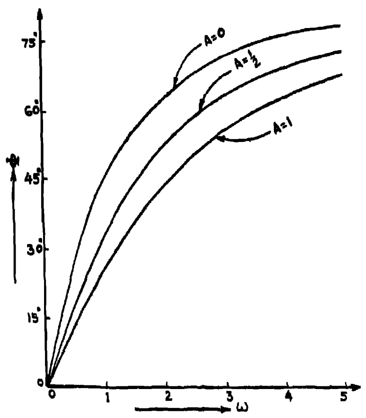

Figure 3. Curves showing $\phi = \tan^{-1} B_i/B_r$ $Vs.$ ω for $A = 0, \frac{1}{2}, 1$

To find the expression for the skin friction, we put the expansions for m, h, b and c into equation 3.3 and get

$$T_0 = \frac{\mu^2 v_0}{L^2 \sigma}\left[\left(1+\frac{\theta_0}{\sigma}\right)+\epsilon e^{i\omega t}\left\{\left[b_r(m_i-\sigma)-b_i m_i+C_r(m_r-h_r\right.\right.\right.$$

$$\left. -C_i(m_i-h_i)+\frac{A}{\omega}\, m_i\left(\frac{\theta_0}{\sigma-\sigma^2}\right)\right]+i\left[b_i(m_r-\sigma)+b_r m_i\right.$$

$$\left.\left.+C_i(m_r-h_r)+C_r(m_i-h_i)-\frac{A}{\omega}(m_r\text{-}1)\left(1+\frac{\theta_0}{\sigma-\sigma^2}\right)\right\}+\ldots\ldots\right] \quad \ldots 3.24$$

$$= \frac{\mu^2 v_0}{L^2 \rho}\left[\left(1+\frac{\theta_0}{\sigma}\right)+\epsilon e^{i\omega t}\left\{M_r+iM_i\right\}+\ldots\ldots\right] \quad \ldots 3.25$$

where m_r, σm_i and other terms are known from equations 3.4 to 3.10 and M_r and M_i are easily known from equations 3.24 and 3.25.

Using the values of m_r, m_i etc. and simplifying, we have

$$M_r = A\left(2-\frac{\theta_0}{\sigma-1}\right)+\omega^2\left[-\frac{5\theta_1}{\sigma^2(\sigma-1)}+\frac{A}{\sigma}\left\{\theta_0\,\frac{25\sigma^2-46\sigma+19}{\sigma(\sigma-1)^3}\right.\right.$$

$$\left.\left.-\frac{(4\sigma+2)(3\sigma-8)}{\sigma(\sigma-1)^2}\right\}\right] \quad ...3.26$$

$$M_i = \frac{A}{\omega\sigma}\,(\theta_0-\sigma^2+\sigma)+\frac{\omega}{\sigma(\sigma-1)}\left[A\left\{\frac{\theta o(\sigma^2+5\sigma-4)}{\sigma(\sigma-1)}\right.\right.$$

$$\left.\left.-2(\sigma-2)\right\}-\theta_1\right] \quad ...3.27$$

Thus from above equations 3.26 and 3.27, we observe, that for sufficiently small values of ω, M_r and M_i are approximately given by first terms of the equations. Equation 3.25 may be written approximately by considering real parts alone,

$$T_0 = \frac{\mu^2 v_0}{L^2\rho}\left\{\left(1+\frac{\theta_0}{\sigma}\right)+\epsilon\,|\,M\,|\,\cos\left(\omega t+\tan^{-1}\frac{M_i}{M_r}\right)\right\} \quad ...3.28$$

where $M = \sqrt{Mr^2+Mi^2}$

$$= \frac{A}{\omega\sigma(\sigma-1)}\sqrt{\sigma^2\omega^2(2\sigma-2-\theta_0)^2+(\sigma-1)^2(\theta_0-\sigma^2+\sigma)^2} \quad 3.29$$

and the sign of $\tan^{-1}\dfrac{M_i}{M_r}$ depends upon the values of ω, θ_0, σ and A.

The variations of M with A for given values of ω has been shown graphically in figure 4 when $\sigma = 3/2$, $\theta_0 = 1$. This has been selected for simplicity of calculations and then a linear relation exists between M and A when ω is given certain value. It is found from the graph that M and A increases together and thus the skin friction increases as the suction velocity increases.

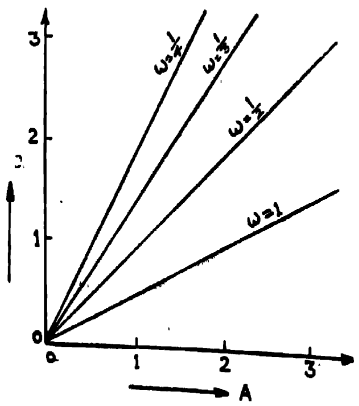

Figure 4. Graph between M and A for $\omega = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, 1$.

Case (ii) if ω is large

If ω^{-2} and higher negative powers of ω are omitted, we have in this case

$$\left.\begin{aligned}
h &\simeq \sqrt{\frac{\omega\sigma}{2}}\,(1+i) \\
m &\simeq \sqrt{\omega\sigma}\,(1+i) \\
b &\simeq \frac{1}{\omega^2}\Big[\,A\sigma^2(\sigma-1)+A\sigma\theta_0\,\Big]+\frac{iA\sigma}{\omega} \\
h_1^2-h-i\omega &\simeq (\sigma-1)\left[\left(\frac{\sigma}{2}+\sqrt{\frac{\omega\sigma}{2}}+i\left(\omega+\sqrt{\frac{\omega\sigma}{2}}\right)\right)\right] \\
c &\simeq \frac{\theta_0\sigma A}{\omega^2(\sigma-1)}+\frac{i\,\theta_1}{\omega(\sigma-1)}
\end{aligned}\right\} \quad \ldots\ 3.30$$

Thus we have

$$F_1 = \frac{A}{\sigma-1}\Big[e^{-\sigma y}-e^{-y}\sqrt{2i\omega\sigma}\Big]+\frac{i}{\omega}\Big[e^{-y}\sqrt{2i\omega\sigma}\Big\{A\left(1+\frac{\theta_0}{(1-\sigma)}\right)$$

$$-A\left(1+\frac{\theta_0}{\sigma-\sigma^2}\right)e^{-y}+\frac{\theta_1}{\sigma-1}\,e^{-y}\sqrt{i\omega\sigma}\Big\}\Big] \quad \ldots\ 3.31$$

$$T_1 = \theta_1 \, e^{-y\sqrt{i\omega\sigma}} + \frac{i\theta_0 \sigma A}{\omega} \left[e^{-\sigma y} - e^{-y\sqrt{i\omega\sigma}} \right] \qquad \ldots 3.32$$

$$\text{and } q = \frac{K\nu_0 \nu^2}{g\beta L} \left\{ \theta_0 \sigma + \epsilon \, e^{i\omega t} \left[\theta_1 \sqrt{i\,\omega\sigma} \right.\right.$$

$$\left.\left. + \frac{i\,\theta_0\,\sigma A}{\omega} \left(\sigma - \sqrt{i\,\omega\sigma} \right) + \ldots \ldots \right\} \right] \qquad \ldots 3.33$$

where, we easily find that q decreases as A increases. However if the real and imaginary parts are separated, we have

$$q = \frac{K\,\nu_0\,\nu^2}{g\,\beta\,L^4} \left[\theta_0 + \sigma + \epsilon \left\{ \theta_1 \sqrt{\omega\,\sigma} \, \cos\left(\omega t + \tfrac{\pi}{4}\right) \right.\right.$$

$$\left.\left. + \frac{\theta_0 \sigma^2 A}{\omega} \cos\left(\omega t + \tfrac{\pi}{2}\right) - \theta_0 \sigma A \sqrt{\frac{\sigma}{\omega}} \cos\left(\omega t + \tfrac{3\pi}{4}\right) \right\} + \ldots \right] \qquad 3.34$$

from which looking at various terms, it is clear that the wall temperature leads by certain angles. The values of F_0 and T_0 are uneffected by changes in ω. If we substitute the equations 3.32 and 3.31 into 2.1 and 2.2, we have expressions for $u(y,t)$ and $T(y,t)$ with the help of equations 2.5 and 2.11.

Now using the equations of set 3.30 into 3.3, we have

$$T_0 = \frac{\mu^2 \nu_0}{L^2 \rho} \left[1 + \frac{\theta_0}{\sigma} + \epsilon e^{i\omega t} \left\{ L_r + i L_i \right\} + \ldots\ldots \right] \qquad \ldots 3.35$$

$$\text{where, } L_r = \bar{\omega}^{1/2} \left[A \left\{ \sigma^{1/2} \left(1 - \frac{\theta_0}{\sigma(\sigma-1)} \right) - \sigma^{3/2} \right.\right.$$

$$\left.\left. - \frac{0.3\theta_1 \sigma^{1/2}}{\sigma - 1} \right] + \omega^{-3/2} A \left[\sigma^{5/2}(\sigma-1) + \sigma^{3/2} \theta_0 \left(1 + \frac{0.3}{\sigma-1} \right) \right]$$

$$-\omega^{-2} \left[A\sigma^3(\sigma-1) + A\sigma^2\theta_0 \right] \qquad \ldots 3.36$$

$$\text{and } L_i = \omega^{-1/2}\left[A\left\{\sigma^{3/2}+\frac{\theta_0\sigma^{-1/2}}{\sigma-1}-\sigma^{-1/2}\right\}+\frac{0.3\,\theta_1\sigma^{1/2}}{\sigma-1}\right]$$

$$+\,\omega^{-1}\left[A\left(1-\sigma^2-\frac{\theta_0}{\sigma(\sigma-1)}\right)\right]+\omega^{-3/2}\left[A\sigma^{3/2}\left\{\frac{0.3\,\theta_0}{\sigma-1}\right.\right.$$

$$\left.\left.+\,\sigma(\sigma-1)+\theta_0\right\}\right] \qquad \ldots 3.37$$

Thus as in previous cases, the phase angle $\tan^{-1} L_i/L_r$ may easily be calculated for given values of parameters. Dividing equations 3.37 and 3.36, we easily see that L_i/L_r becomes -45° when ω is very large. Thus for very large frequency of oscillations, the skin friction lags behind the wall fluctuations by 45°. If $A = 0$, *i. e.* steady suction we also have the same conclusions.

We see that L_r and L_i are important factors and thus its variations with ω has been shown graphically in figure 5 when $\sigma=0.64$ $\theta_0 = \theta_1 = 1$ and $A = 0, \frac{1}{2}, 1$.

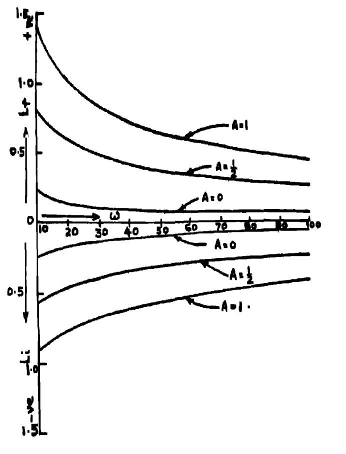

Figure 5. Curves showing L_r, and L_i against ω for $A = 0, \frac{1}{2}, 1$.

From the graph, we find that L_i is always negative and thus ths skin-friction lags behind the wall fluctuations. The value of the ratio of L_i and L_r may easily be calculated from the figure for given A at various values of ω. As an example for $\omega = 10$, we have from figure 5, that $| L_i/L_r | \leqslant 1$ if $A \geqslant 0$. Thus the phase angle becomes less than 45°.

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